Fuzzy Logic Control

Type of Fuzzy Controllers:

- Mamdani
- TSK (Takagi Sugeno Kang)
- Tsukamoto

Fuzzy Control Systems

Mamdani

Fuzzy models

Mamdani Fuzzy models

- The most commonly used fuzzy inference technique is the so-called Mamdani method.

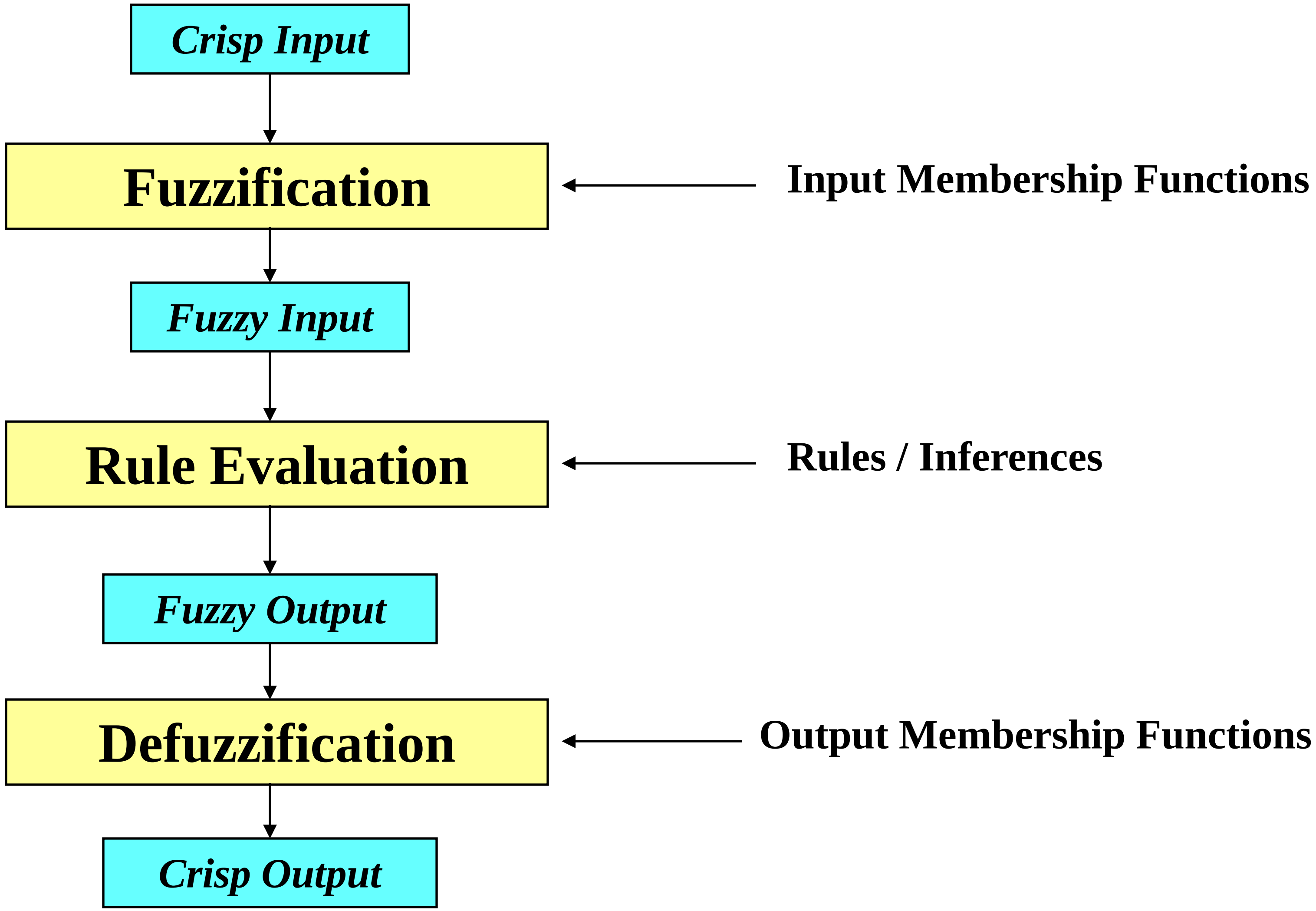
- In 1975, Professor **Ebrahim Mamdani** of London University built one of the first fuzzy systems to control a steam engine and boiler combination.
- Original Goal: Control a steam engine & boiler combination by a set of linguistic control rules obtained from experienced human operators.

Mamdani fuzzy inference

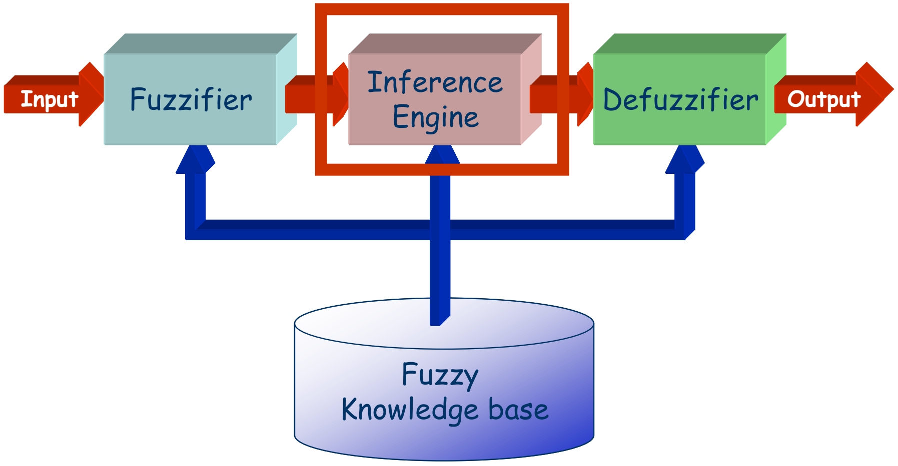
The Mamdani-style fuzzy inference process is performed in four steps:

1. Fuzzification of the input variables,
2. Rule evaluation;
3. Aggregation of the rule outputs, and finally
4. De-fuzzification.

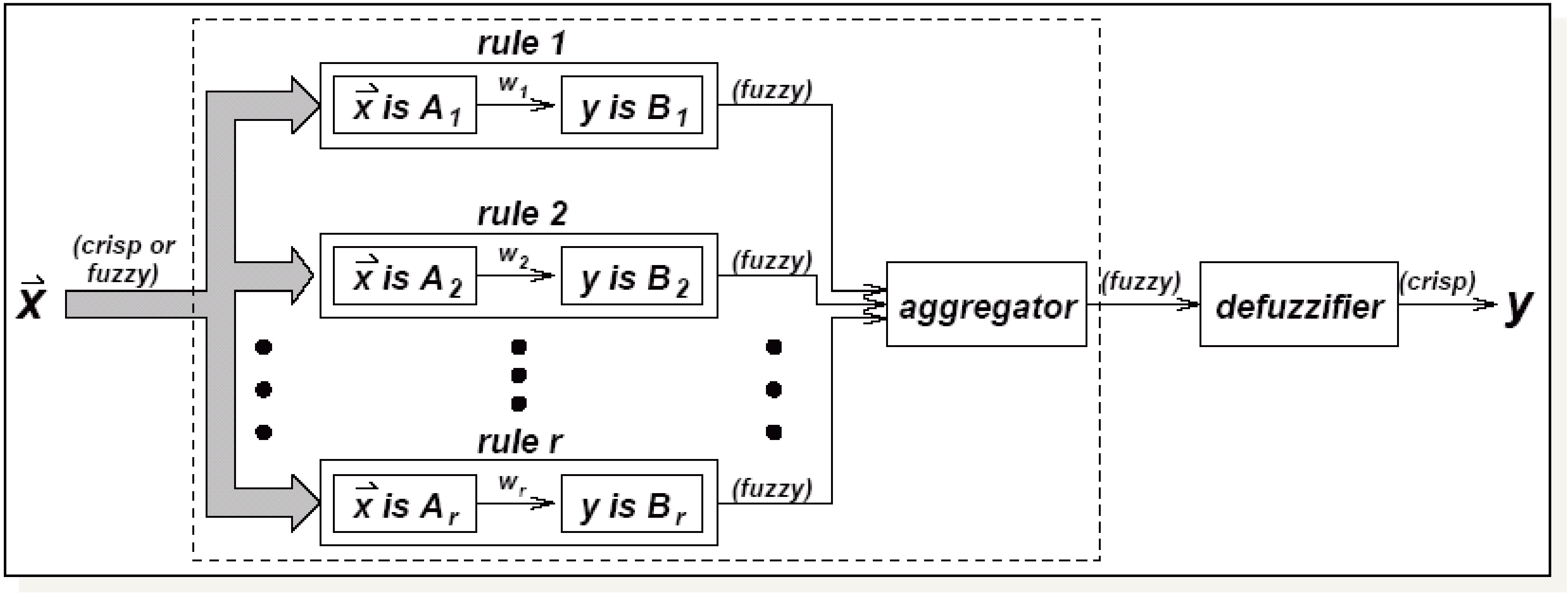
Operation of Fuzzy System



Inference Engine



Using If-Then type fuzzy rules converts the fuzzy input to the fuzzy output.

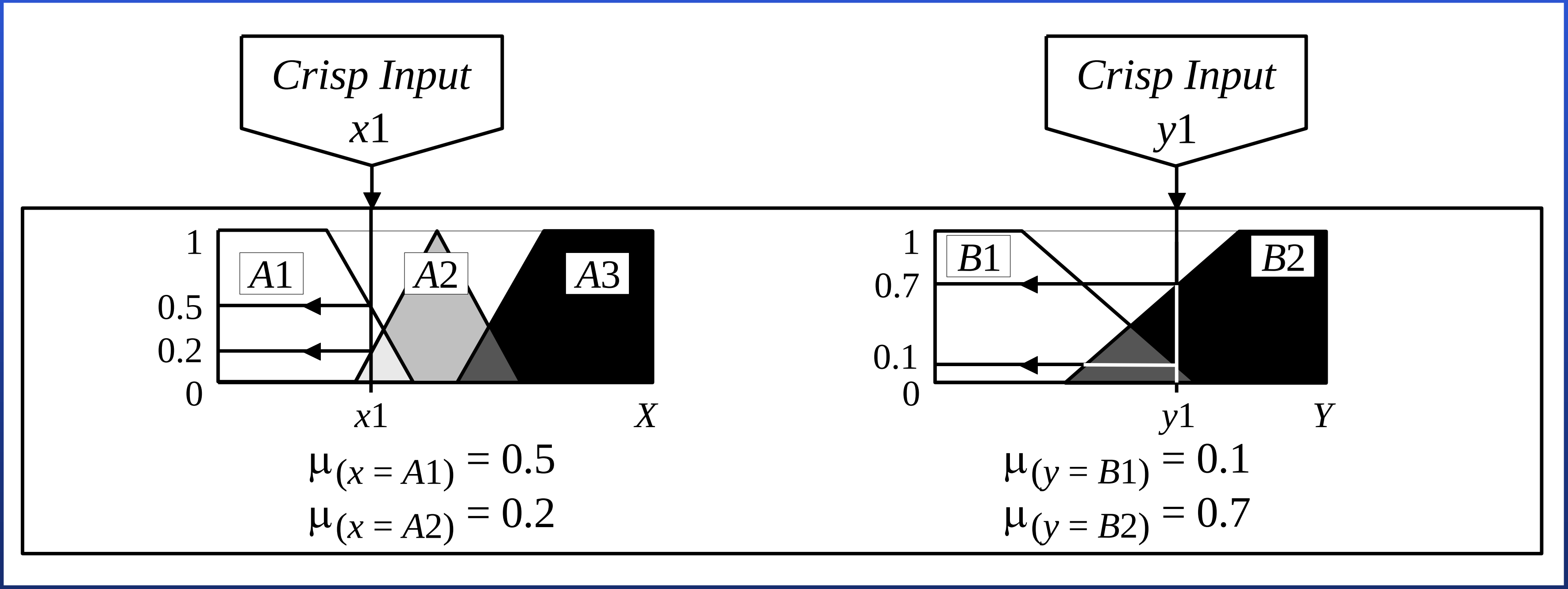


We examine a simple two-input one-output problem that includes three rules:

Rule: 1	Rule: 1
IF *x* is *A*3	IF *project_funding* is *adequate*
OR *y* is *B*1	OR *project_staffing* is *small*
THEN *z* is *C*1	THEN *risk* is *low*
Rule: 2	Rule: 2
IF *x* is *A*2	IF *project_funding* is *marginal*
AND *y* is *B*2	AND *project_staffing* is *large*
THEN *z* is *C*2	THEN *risk* is *normal*
Rule: 3	Rule: 3
IF *x* is *A*1	IF *project_funding* is *inadequate*
THEN *z* is *C*3	THEN *risk* is *high*

Step 1: Fuzzification

- Take the crisp inputs, $x1$ and $y1$ (*project funding* and *project staffing*)
- Determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



project funding *project staffing*

Step 2: Rule Evaluation

- take the fuzzified inputs, $\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$

- apply them to the antecedents of the fuzzy rules.

- If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation. This number (the truth value) is then applied to the consequent membership function.

Step 2: Rule Evaluation

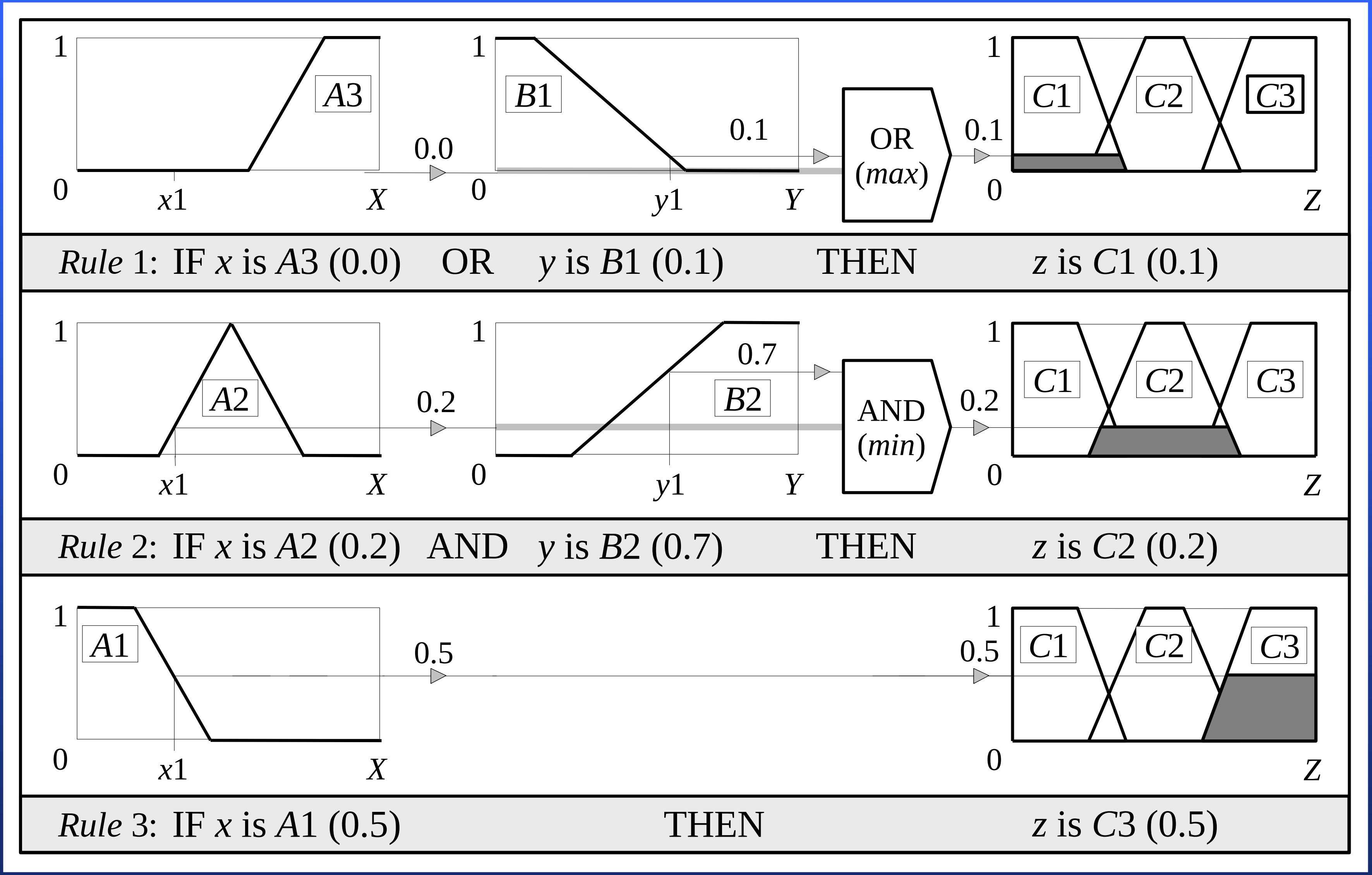
To evaluate the disjunction of the rule antecedents, we use the **OR fuzzy operation**. Typically, fuzzy expert systems make use of the classical fuzzy operation **union**:

$$\mu_{A \cup B}(x) = max\, [\mu_A(x),\, \mu_B(x)]$$

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the **AND fuzzy operation intersection**:

$$\mu_{A \cap B}(x) = min\, [\mu_A(x),\, \mu_B(x)]$$

Mamdani-style rule evaluation



Step 3: Aggregation of The Rule Outputs

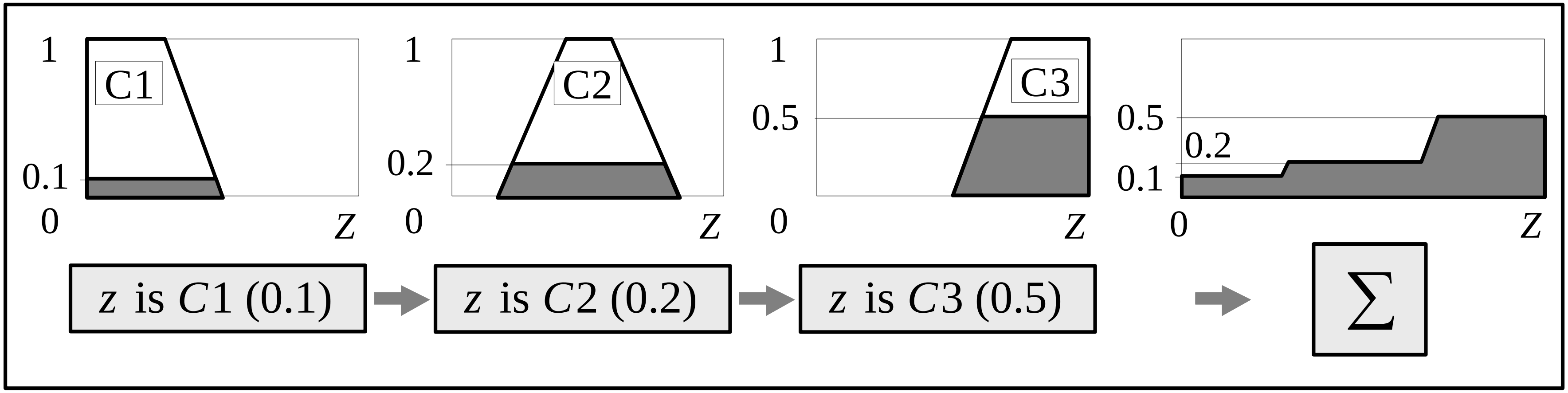
- Aggregation is the process of unification of the outputs of all rules.

- We take the membership functions of all rule consequents previously clipped or scaled and combine them into a single fuzzy set.

- There are several defuzzification methods, but probably the most popular one is the **centroid technique**.
- It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this **centre of gravity (COG)** can be expressed as:

$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

Aggregation of the rule outputs



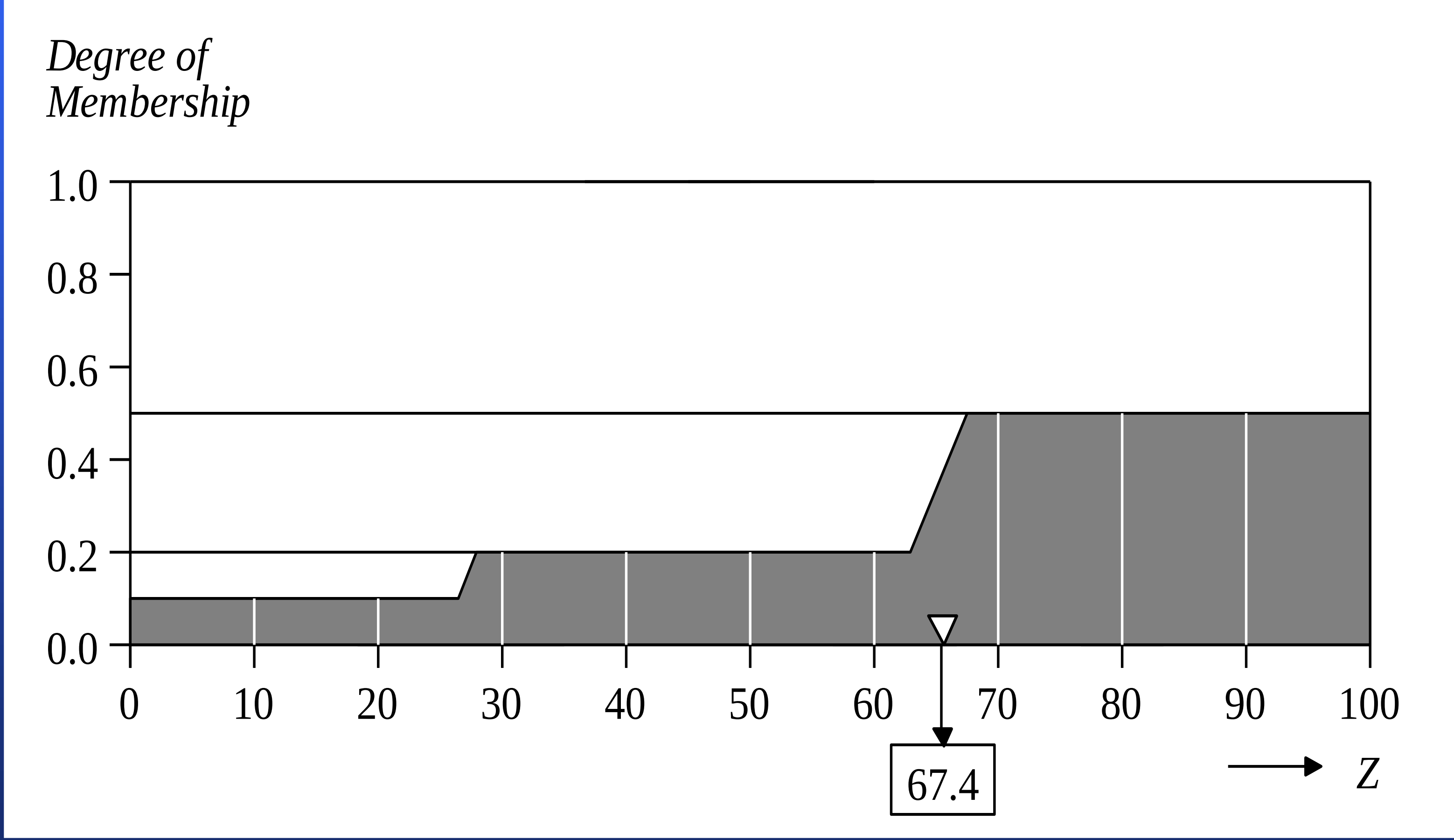
Step 4: Defuzzification

- Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.

- The input for the defuzzification process is the aggregated output fuzzy set and the output is a single number.

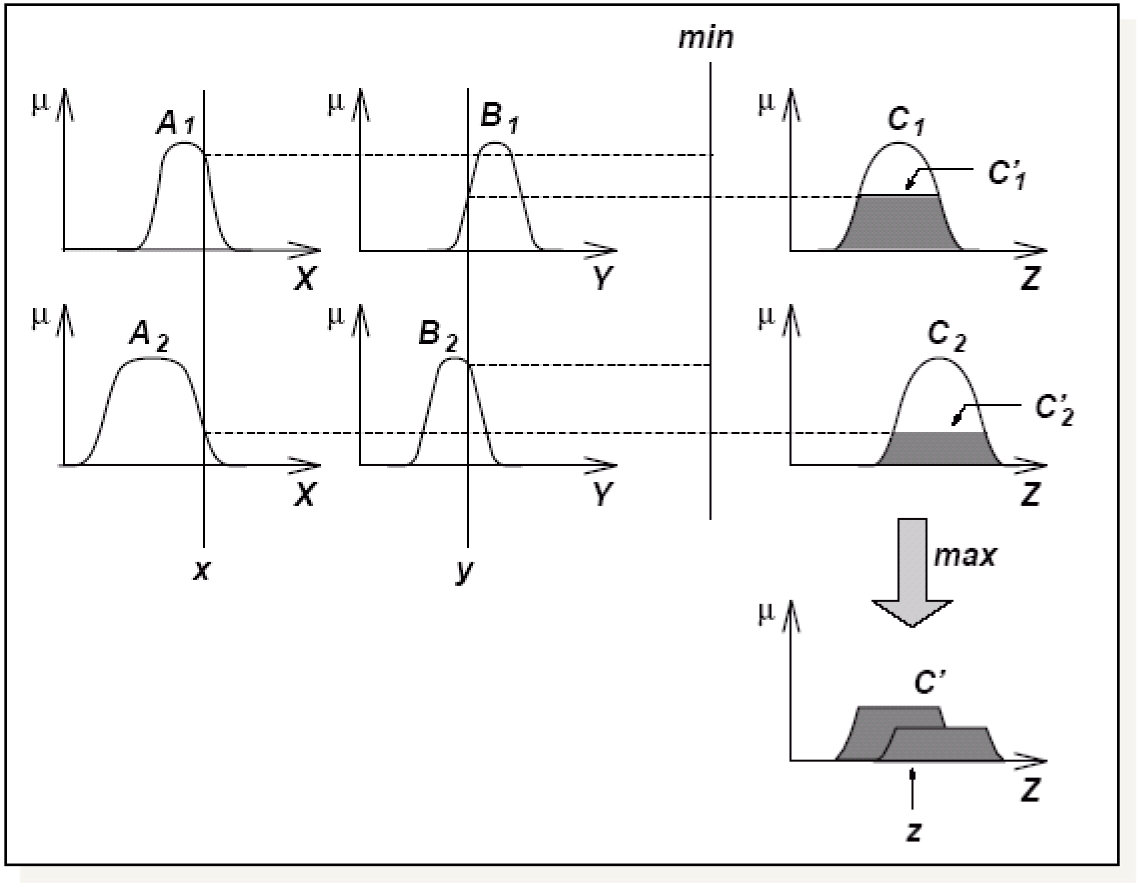
Centre of gravity (COG):

$$COG = \frac{(0+10+20)\times 0.1+(30+40+50+60)\times 0.2+(70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$



Max-Min Composition is used.

The Reasoning Scheme



Fuzzy Control Systems

Sugeno

Fuzzy Models

Sugeno Fuzzy Models

- Also known as TSK fuzzy model
  - Takagi, Sugeno & Kang, 1985

- Goal: Generation of fuzzy rules from a given input-output data set.

Sugeno Fuzzy Control

- Mamdani-style inference, requires to find the centroid of a two-dimensional shape
  - by integrating across a continuously varying function.
  - In general, this process is not computationally efficient.
- **Michio Sugeno** suggested to use a single spike, a *singleton*, as the membership function of the rule consequent.
- A **fuzzy singleton**, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

- Sugeno-style fuzzy inference is very similar to the Mamdani method.
- Sugeno changed only a rule consequent. Instead of a fuzzy set, he used a mathematical function of the input variable.
- The format of the **Sugeno-style fuzzy rule** is

IF x is A AND y is B

THEN z is $f(x, y)$

- where x, y and z are linguistic variables
- A and B are fuzzy sets on universe of discourses X and Y
- $f(x, y)$ is a mathematical function

The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

IF ***x*** **is** ***A*** **AND** ***y*** **is** ***B***
THEN ***z*** **is** ***k***

where *k* is a constant.

- In this case, the output of each fuzzy rule is constant.
- All consequent membership functions are represented by singleton spikes.

Fuzzy Rules of TSK Model

If x is A and y is B then $z = f(x, y)$

Fuzzy Sets

Crisp Function

$f(x, y)$ is very often a polynomial function w.r.t. x and y.

Examples

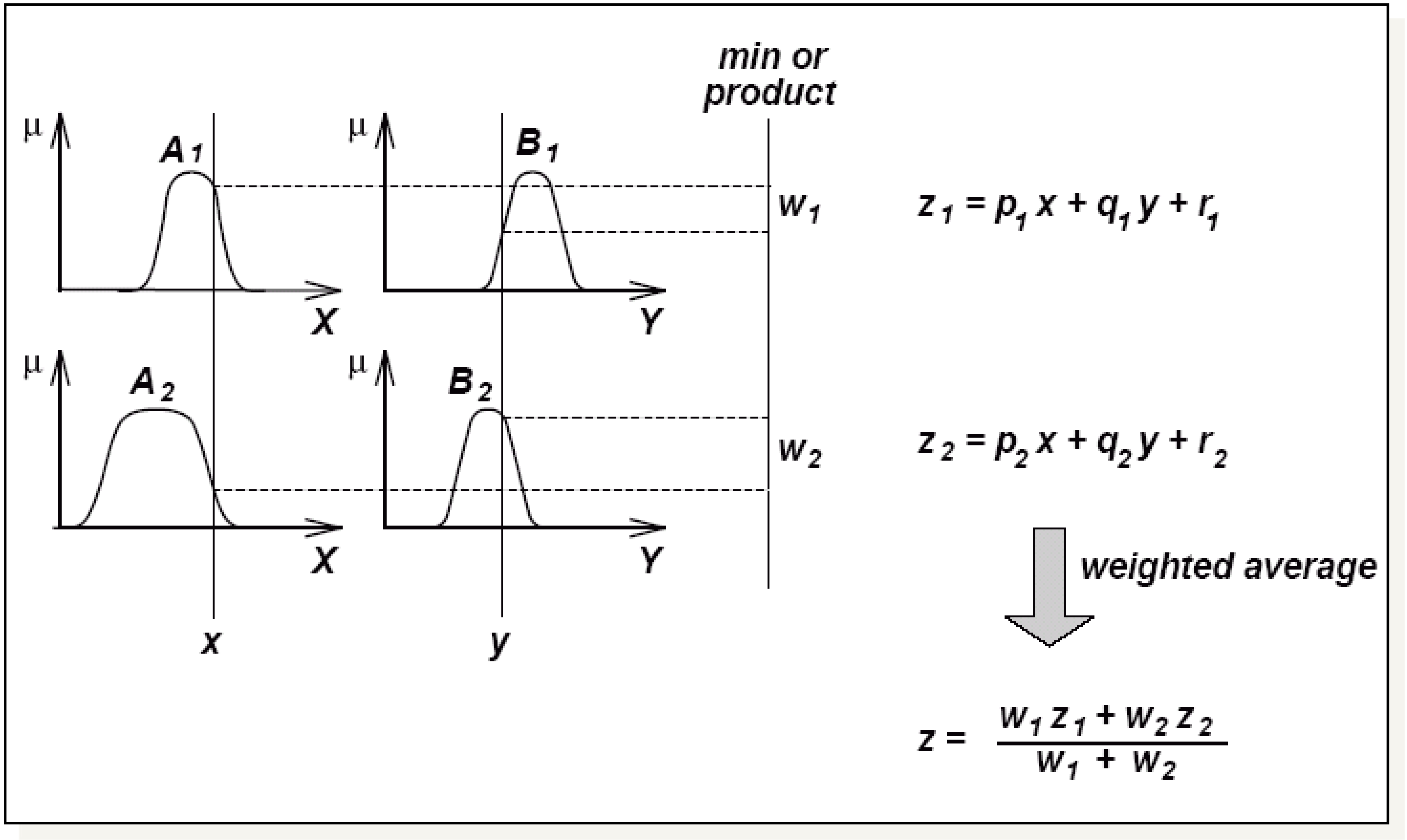
R1: if X is small and Y is small then $z = -x + y + 1$

R2: if X is small and Y is large then $z = -y + 3$

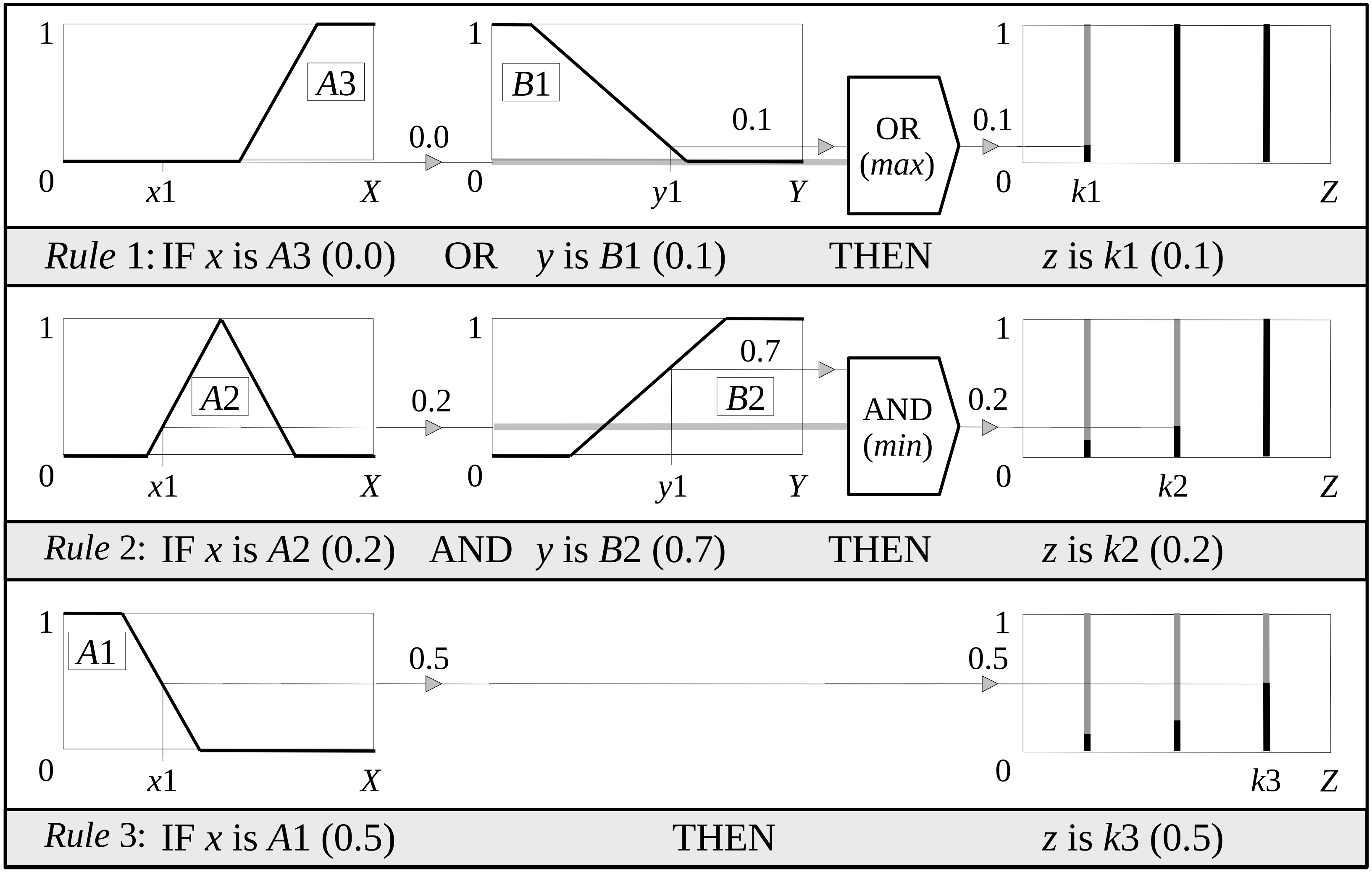
R3: if X is large and Y is small then $z = -x + 3$

R4: if X is large and Y is large then $z = x + y + 2$

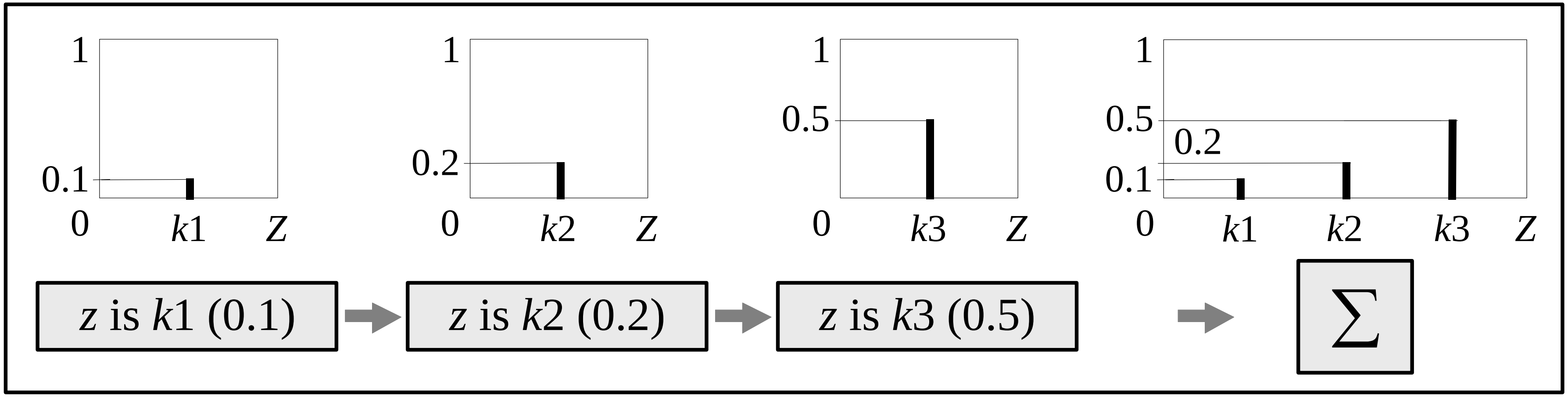
The Reasoning Scheme



Sugeno-style rule evaluation



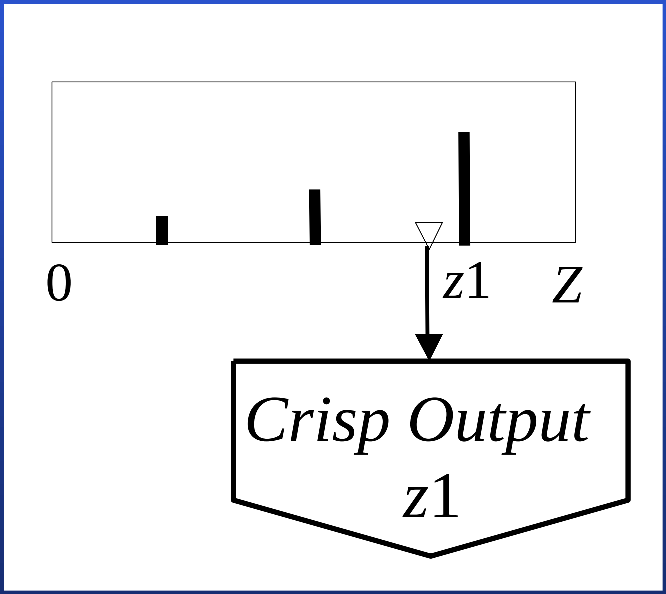
Sugeno-style aggregation of the rule outputs



Weighted average (WA):

$$WA = \frac{\mu(k1)\times k1 + \mu(k2)\times k2 + \mu(k3)\times k3}{\mu(k1)+\mu(k2)+\mu(k3)} = \frac{0.1\times 20 + 0.2\times 50 + 0.5\times 80}{0.1+0.2+0.5} = 65$$

Sugeno-style defuzzification



Tsukamoto Fuzzy Model

The consequent of each fuzzy if-then rule:

- a fuzzy set with a monotonical MF.
- Overall output: the weighted average of each rule's output.
- No defuzzification.
- Not as transparent as mamdani's or Sugeno's fuzzy model.
- Not follow strictly the compositional rule of inference: the output is always crisp.

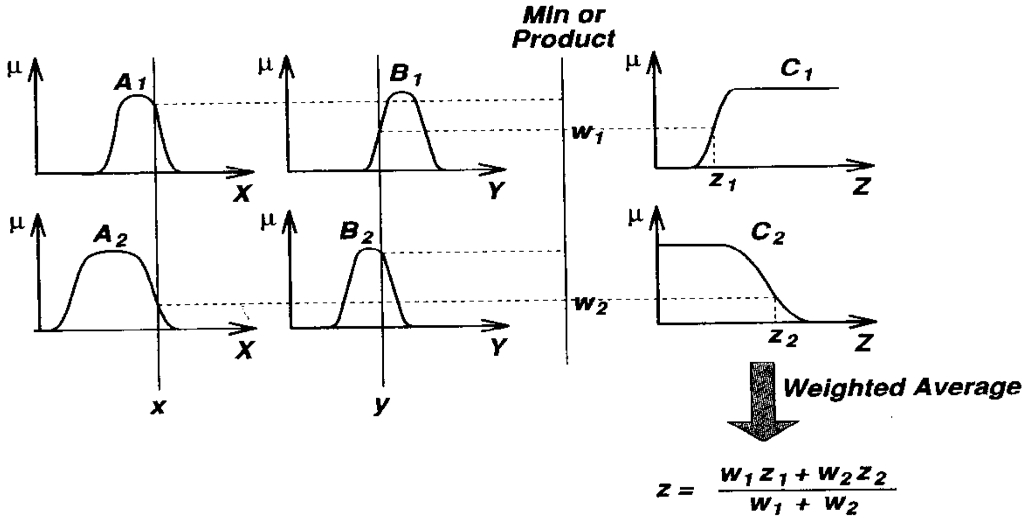
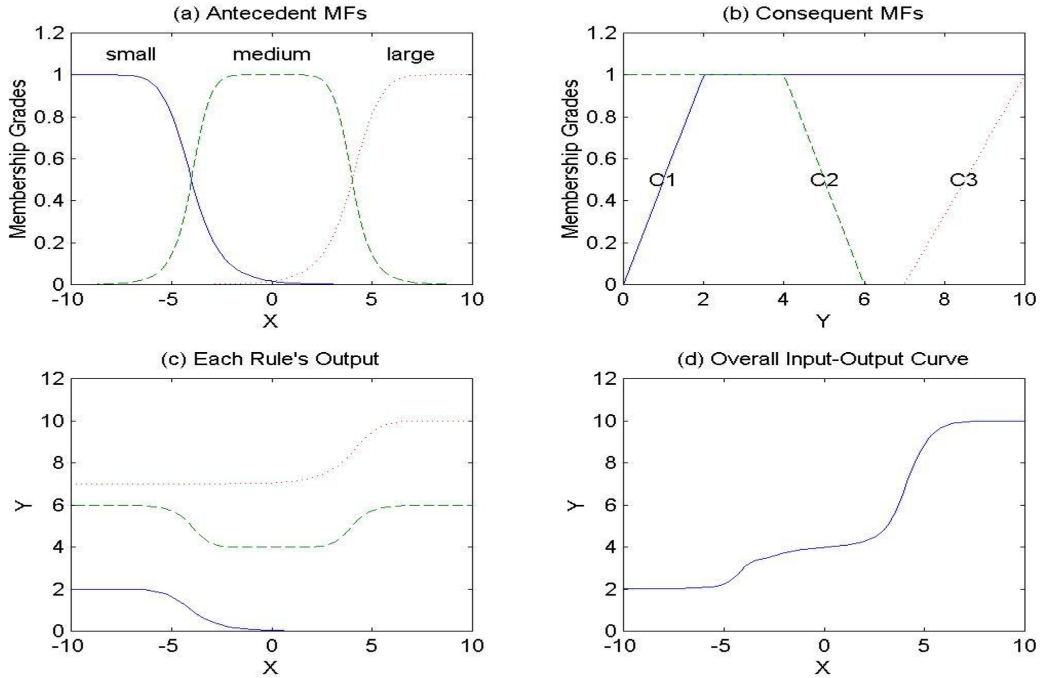


Figure 4.11. *The Tsukamoto fuzzy model.*

Example: Tsukamoto Fuzzy Model

Single-input Tsukamoto fuzzy model

If X is small then Y is C_1.
If X is medium then Y is C_2.
If X is large then Y is C_3.



Tuning Fuzzy Systems

1. Review model input and output variables, and if required redefine their ranges.

2. Review the fuzzy sets, and if required define additional sets on the universe of discourse.
   - The use of wide fuzzy sets may cause the fuzzy system to perform roughly.

3. Provide sufficient overlap between neighbouring sets.
   - It is suggested that triangle-to-triangle and trapezoid-to-triangle fuzzy sets should overlap between 25% to 50% of their bases.

4. Review the existing rules, and if required add new rules to the rule base.

5. Adjust the rule execution weights. Most fuzzy logic tools allow control of the importance of rules by changing a weight multiplier.

6. Revise shapes of the fuzzy sets. In most cases, fuzzy systems are highly tolerant of a shape approximation.

Steps in Designing a Fuzzy Logic Control System

1. Identify the system input variables, their ranges, and membership functions.
2. Identify the output variables, their ranges, and membership functions.
3. Identify the rules that describe the relations of the inputs to the outputs.
4. Determine the de-fuzzifier method of combining fuzzy rules into system outputs.